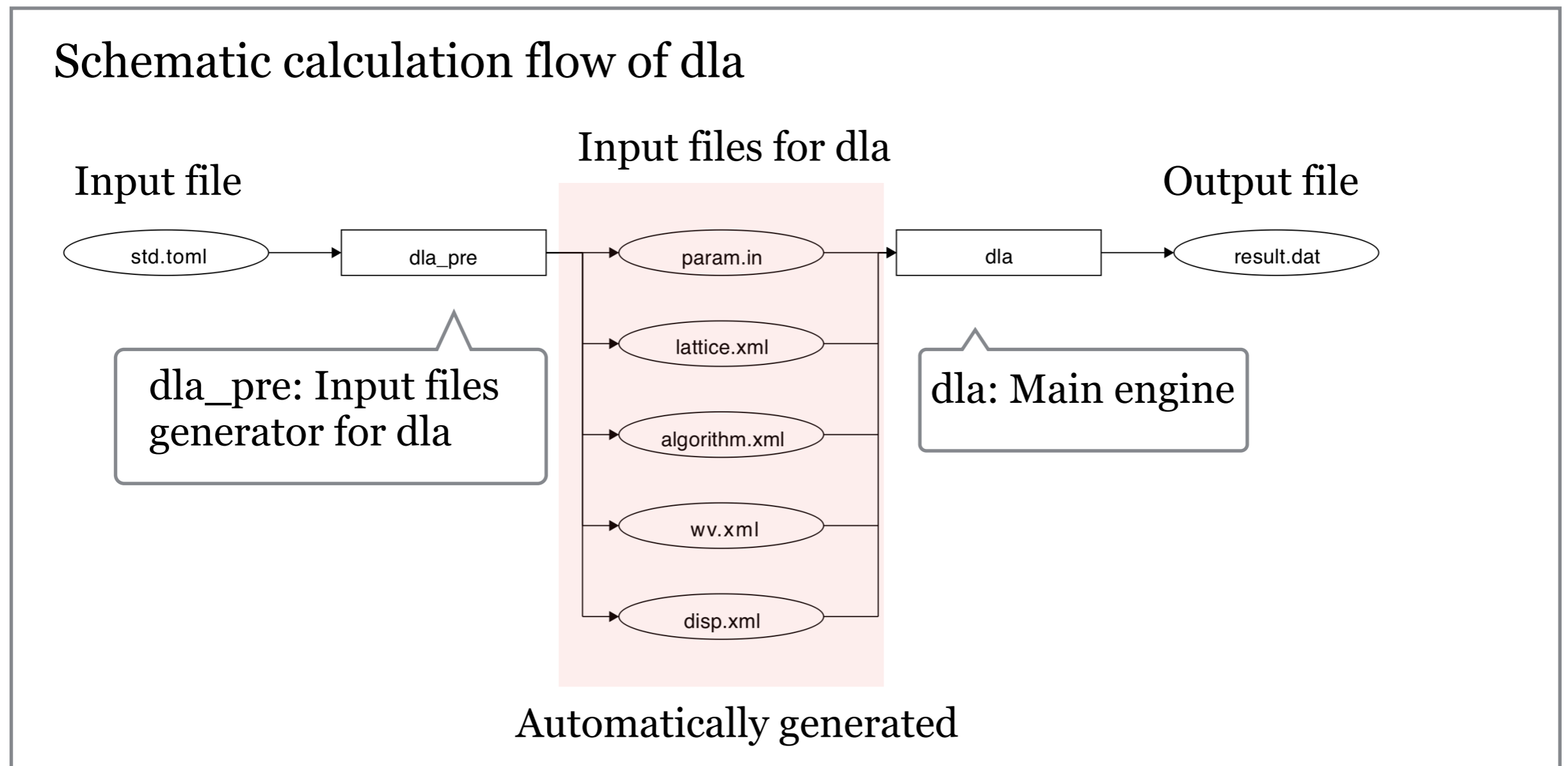


The simple mode of DSQSS/DLA

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2019-06-06 @ ISSP

The simple mode of DSQSS/dla

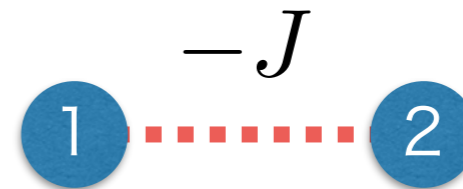
- Users can simulate of a predefined model on a predefined lattice from one text file (std.toml file).



Tutorial: Heisenberg dimer (1)

- **Sample of simple mode file (sample/dla/01_spindimer/std.toml)**
- **S=1/2 antiferromagnetic Heisenberg dimer**

$$\mathcal{H} = -J \mathbf{S}_1 \cdot \mathbf{S}_2$$



```
[hamiltonian]
model = "spin"
M = 1           # S=1/2
Jz = -1.0       # coupling constant, negative for AF
Jxy = -1.0      # coupling constant, negative for AF
h = 0.0         # magnetic field
```

```
[lattice]
lattice = "hypercubic" # hypercubic, periodic
dim = 1                # dimension
L = 2                  # number of sites along each direction
bc = false             # open boundary
```

[hamiltonian] section

Specify information of
Hamiltonian

[lattice] section

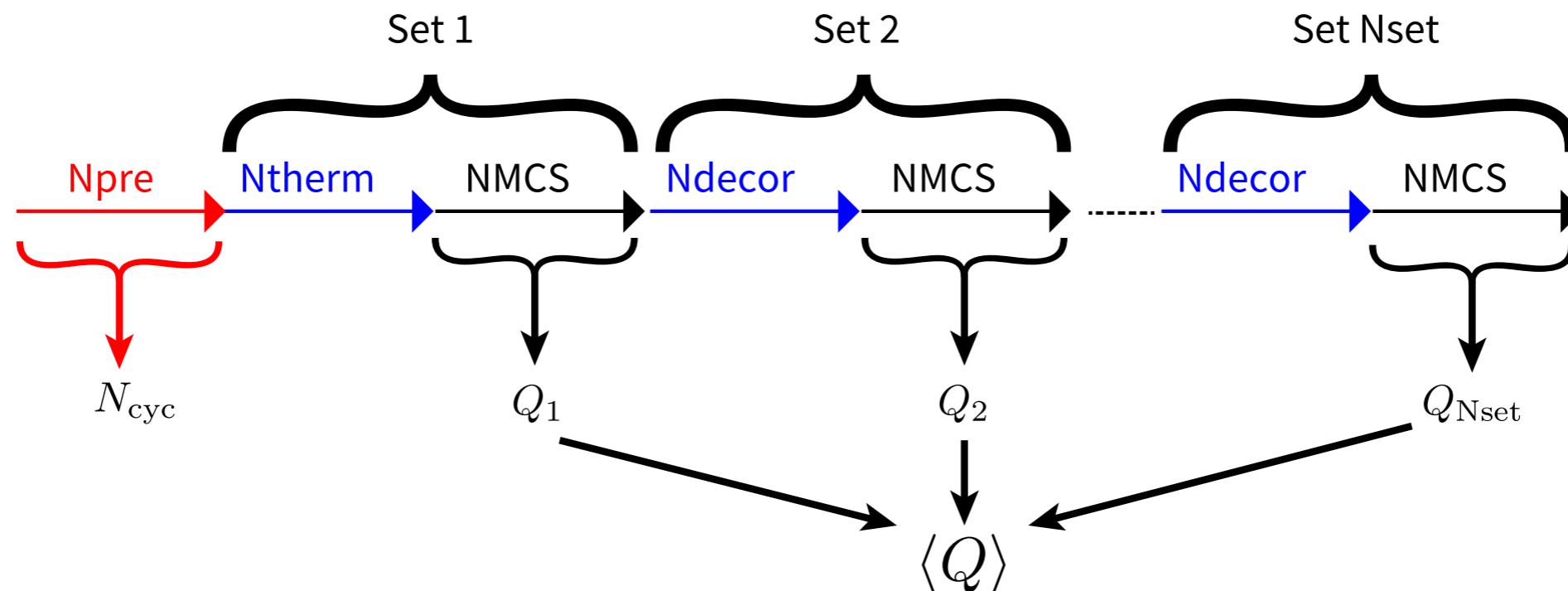
Specify information of lattice

Tutorial: Heisenberg dimer (2)

- **Sample of simple mode file (sample/dla/01_spindimer/std.toml)**

```
[parameter]
beta = 100      # inverse temperature
nset = 5        # set of Monte Carlo sweeps
npre = 10       # MCSteps to estimate hyperparameter
ntherm = 10     # MCSweeps for thermalization
nmcs = 100     # MCSweeps for measurement
seed = 31415    # seed of RNG
```

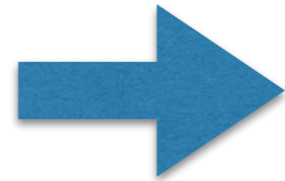
[parameter] section
Specify simulation parameters



Tutorial: Heisenberg dimer (3)

- **Run the dla_pre**

```
$ dla_pre std.toml
```



**param.in, lattice.xml,
algorithm.xml**

output

- **Run the dla**

```
$ dla param.in
```

Standard output (log)

Random number parallelization

```
$ mpiexec -np 4 dla param.in
```

```
>>> The program is being run with MPI mode.( N_PROC = 1 )
+++++++ input data ++++++++
RUNTYPE = 0
...
+++++++ input data ++++++++
Determining hyperparameter NCYC : 51
Start main calculation.
1 / 5 done. [Elapsed: 0.014687 sec. ETR: 0.058748 sec.]
2 / 5 done. [Elapsed: 0.195772 sec. ETR: 0.293658 sec.]
3 / 5 done. [Elapsed: 0.333755 sec. ETR: 0.222503 sec.]
4 / 5 done. [Elapsed: 0.471529 sec. ETR: 0.117882 sec.]
5 / 5 done. [Elapsed: 0.616055 sec. ETR: 0 sec.]
```

ETR: Estimated Time Remaining

Tutorial: Heisenberg dimer (4)

Output file: sample.log

```
IN_PROC = 1
P D      =      1
P L      =      2
...
P SIMULATIONTIME = 0.000000
R sign = 1.000000000e+00 0.000000000e+00
R anv = 2.493000000e+01 2.96344394e-01
R ene = -3.743000000e-01 2.96344394e-03
R spe = 8.164000000e-02 1.47017825e+00
R som = 8.164000000e+00 1.47017825e+02
R len = 4.00127485e+00 2.92088648e-02
R xmx = 1.00031871e-02 7.30221620e-05
...
R time = 1.37698000e-06 1.20496230e-07
I [the maximum number of segments] = 165
I [the maximum number of vertices] = 84
I [the maximum number of reg. vertex info.] = 1
```

P <name> = <value>

Parameters read from the input files.

R <name> = <mean> <error>

Results of observables.

<mean> : the expected value

<error> : the statistical error of <mean>.

I <text> = <value>

Other information.

Main Results are written in **[R]** section.

- **Analyze the result**

```
$ grep ene sample.log
```

```
R ene = -3.743000000e-01 2.96344394e-03
```

Ground state energy

-3.75 (exact)

Tutorial: Heisenberg dimer (5)

Main Results 1/3

sign

The sign of the weights.

$$\sum_i W_i / \sum_i |W_i|$$

anv

The mean number of the vertices.

$$\frac{\langle N_v \rangle}{N_s}$$

ene

The energy density (energy per site)

$$\epsilon \equiv \frac{1}{N_s} (E_0 - T \langle N_v \rangle)$$

spe

The specific heat

$$C_V \equiv \frac{\partial \epsilon}{\partial T}$$

Tutorial: Heisenberg dimer (6)

Main Results 2/3

xmx

The transverse susceptibility

amzu

The “magnetization” (uniform, $\tau = 0$).

$$\langle m^z \rangle, \text{ where } m^z \equiv \frac{1}{N_s} \sum_i^{N_s} M_i^z$$

bmzu

The “magnetization” (uniform, average over τ). $\langle \tilde{m}^z \rangle$.

smzu

The structure factor (uniform).

$$S^{zz}(\vec{k} = 0) \equiv \frac{1}{N_s} \sum_{i,j} e^{i\vec{k} \cdot (\vec{r}_i - \vec{r}_j)} [\langle M_i^z M_j^z \rangle - \langle M_i^z \rangle \langle M_j^z \rangle] \Big|_{\vec{k}=0} = N_s [\langle (m^z)^2 \rangle - \langle m^z \rangle^2]$$

xmzu

The longitudinal susceptibility (uniform).

$$\chi^{zz}(\vec{k} = 0, \omega = 0) \equiv \frac{\partial \langle \tilde{m}^z \rangle}{\partial h} = \beta N_s [\langle (\tilde{m}^z)^2 \rangle - \langle \tilde{m}^z \rangle^2]$$

Tutorial: Heisenberg dimer (7)

Main Results 3/3

amzsk

The “magnetization” (“staggered”, $\tau = 0$)

$$\langle m_s^z \rangle \text{ where } m_K^z \equiv \frac{1}{N_s} \sum_i^{N_s} M_i^z \cos(\vec{k} \cdot \vec{r}_i).$$

K is an index of wavevector k specified in the wavevector XML file.

bmzu

The “magnetization” (“staggered”, average over τ). $\langle \tilde{m}_K^z \rangle$.

smzs

The structure factor (“staggered”).

$$S^{zz}(\vec{k}) = N_s \left[\langle (m_K^z)^2 \rangle - \langle m_K^z \rangle^2 \right]$$

xmzs

The longitudinal susceptibility (“staggered”).

$$\chi^{zz}(\vec{k}, \omega = 0) = \beta N_s \left[\langle (\tilde{m}_K^z)^2 \rangle - \langle \tilde{m}_K^z \rangle^2 \right]$$

Tutorial: Magnetic Susceptibility of antiferromagnetic spin chains (1)

- **sample/dla/o2_spinchain**
- **S=1/2, 1 antiferromagnetic spin chains**

$$\mathcal{H} = -J \sum_{i=1}^{30} \mathbf{S}_i \cdot \mathbf{S}_{i+1}$$

```
[hamiltonian]
model = "spin"
M = 1          # S=1/2
Jz = -1.0      # coupling constant, negative for AF
Jxy = -1.0     # coupling constant, negative for AF
h = 0.0        # magnetic field

[lattice]
lattice = "hypercubic" # hypercubic, periodic
dim = 1         # dimension
L = 30          # number of sites along each direction
```

For S=1, M = 2

- **Calculate magnetic susceptibility at each beta**

Tutorial: Magnetic Susceptibility of antiferromagnetic spin chains (2)

- Sample script (exec.py)

```
import subprocess
from dsqss.dla_pre import dla_pre
from dsqss.result import Results
```

```
L = 30
```

```
lattice = {"lattice": "hypercubic", "dim": 1, "L": L}
```

```
hamiltonian = {"model": "spin", "Jz": -1, "Jxy": -1}
```

```
parameter = {"nset": 5, "ntherm": 1000, "ndecor": 1000, "nmcs": 1000}
```

Set parameters

```
name = "xmzu"
```

```
Ms = [1, 2]
```

```
Ts = [0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.25, 1.5, 1.75, 2.0]
```

```
for M in Ms:
```

```
    output = open("{0}_{1}.dat".format(name, M), "w")
```

```
    for i, T in enumerate(Ts):
```

```
        ofile = "res_{0}_{1}.dat".format(M, i)
```

```
        pfile = "param_{0}_{1}.in".format(M, i)
```

```
        hamiltonian["M"] = M
```

```
        parameter["beta"] = 1.0 / T
```

Change beta

```
        parameter["outfile"] = ofile
```

```
        dla_pre(
```

```
            {"parameter": parameter, "hamiltonian": hamiltonian, "lattice": lattice},
```

```
            pfile,
```

```
        )
```

Run dla_pre

```
        cmd = ["dla", "param_{0}_{1}.in".format(M, i)]
```

```
        subprocess.call(cmd)
```

Run dla

```
        res = Results(ofile)
```

```
        output.write("{} {} \n".format(T, res.to_str(name)))
```

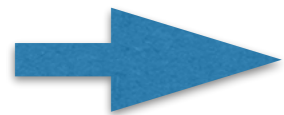
Read and output results

```
    output.close()
```

Tutorial: Magnetic Susceptibility of antiferromagnetic spin chains (3)

- **Run script**

```
$ source $DSQSS_INSTALL_DIR/share/dsqss/dsqssvars-2.0.0.sh  
$ python exec.py
```

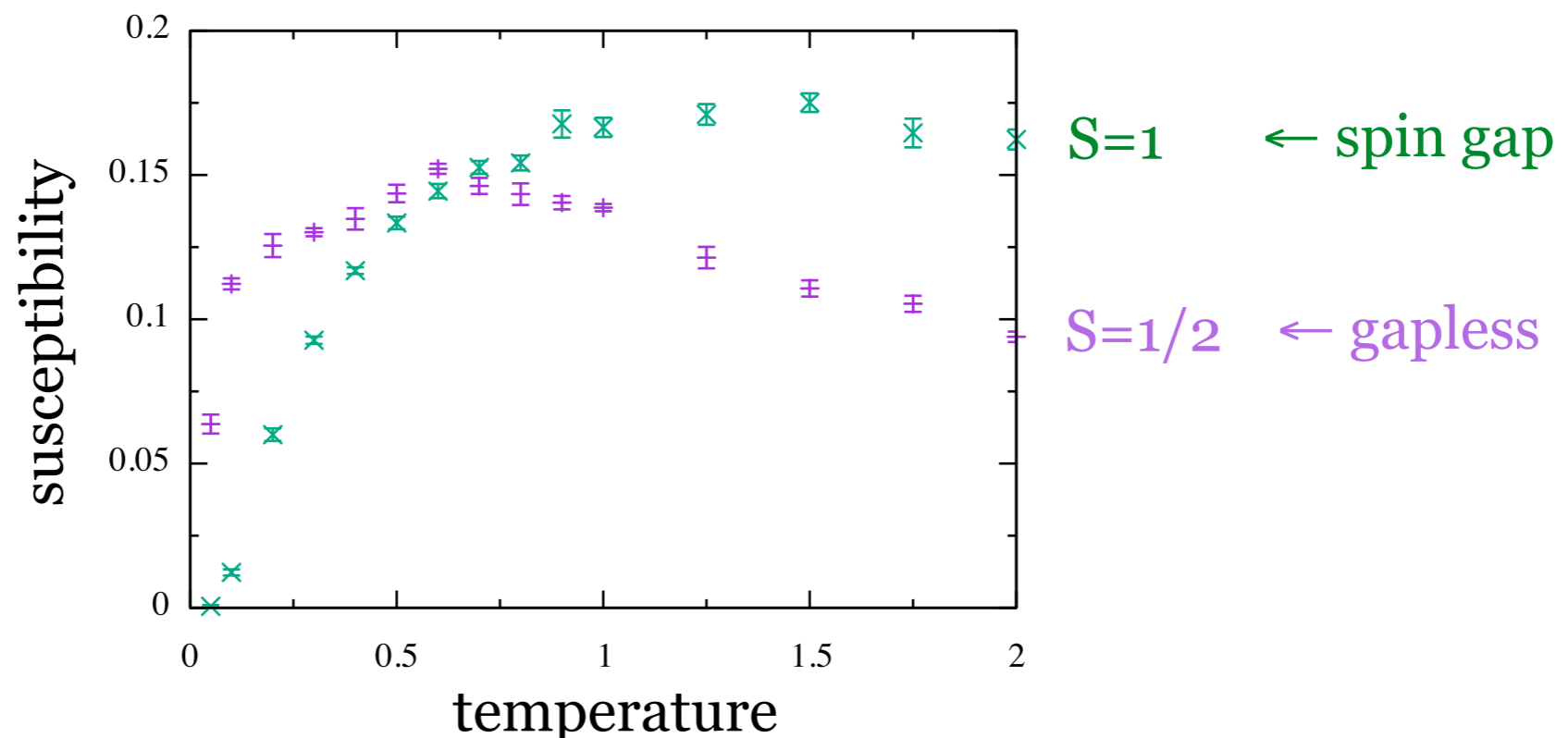


xmzu_1.dat (S=1/2), xmzu_2.dat (S=1)

- **Plot results**

```
$ gnuplot
```

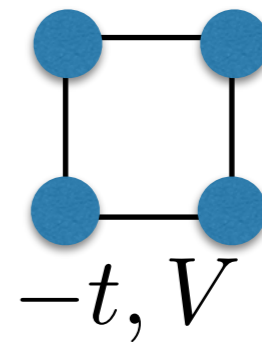
```
$ plot "./xmzu_1.dat" using 1:2:3 with errorbar, "./xmzu_2.dat" using 1:2:3  
with errorbar
```



Tutorial: Number density of the hardcore Bosons on a square lattice (1)

- **sample/dla/03_bosesquare**
Hardcore Bose-Hubbard model with the nearest neighbor repulsive on a 8×8 square lattice

$$\mathcal{H} = -t \sum_{\langle i,j \rangle} \left[b_i^\dagger b_j + b_j^\dagger b_i \right] + \sum_{\langle i,j \rangle} V n_i n_j$$



[hamiltonian]

model = "boson"

M = 1 # The cutoff of the number of particles on a site

t = 1 # The hopping parameter.

V = 3 # The onsite interaction.

[lattice]

lattice = "hypercubic" # hypercubic, periodic

dim = 2 # dimension

L = [8, 8] # number of sites along each direction

Tutorial: Number density of the hardcore Bosons on a square lattice (2)

- **Sample script (exec.py)**

```
import subprocess
```

```
from dsqss.dla_pre import dla_pre  
from dsqss.result import Results
```

```
V = 3  
L = [8, 8]  
beta = 10.0
```

```
lattice = {"lattice": "hypercubic", "dim": 2, "L": L}  
hamiltonian = {"model": "boson", "t": 1, "V": V, "M": 1}  
parameter = {"beta": beta, "nset": 4, "ntherm": 100, "ndecor": 100, "nmcs": 100}
```

Set parameters

```
name = "amzu"  
mus = [-4.0, -2.0, 0.0, 2.0, 2.5, 3.0, 6.0, 9.0, 9.5, 10.0, 12.0, 14.0]
```

```
output = open("{} .dat".format(name), "w")
```

```
for i, mu in enumerate(mus):
```

```
    ofile = "res_{}.dat".format(i)
```

```
    pfile = "param_{}.in".format(i)
```

```
    hamiltonian["mu"] = mu
```

```
    parameter["outfile"] = ofile
```

Change chemical potential

```
    dla_pre(
```

```
        {"parameter": parameter, "hamiltonian": hamiltonian, "lattice": lattice}, pfile
```

```
    )
```

Run dla_pre

```
    cmd = ["dla", pfile]
```

```
    subprocess.call(cmd)
```

Run dla

```
    res = Results(ofile)
```

```
    output.write("{} {} \n".format(mu, res.to_str(name)))
```

Read and output results

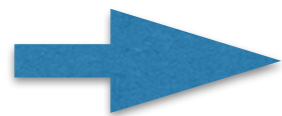
```
output.close()
```

Tutorial: Number density of the hardcore Bosons on a square lattice (3)

- **Sample script (exec.py)**

- **Run script**

```
$ source $DSQSS_INSTALL_DIR/share/dsqss/dsqssvars-2.0.0.sh  
$ python exec.py
```

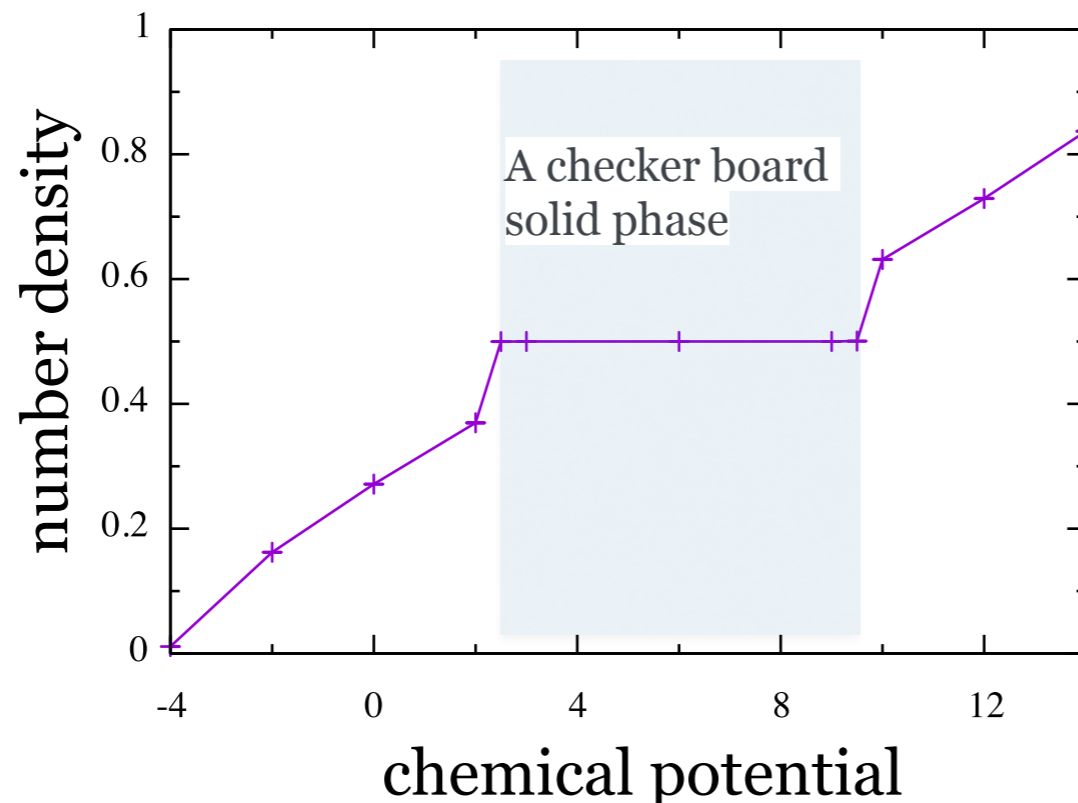


amzu.dat

- **Plot results**

```
$ gnuplot
```

```
$ plot "./zmzu.dat" using 1:2:3 with errorbar, "" using 1:2 w l lc 1
```



Tutorial: Number density of the hardcore Bosons on a square lattice (4)

- Calculate structure factors
 - Add following keywords.

```
[parameter]
wvfile = "wave.out" # A wavevector XML file.
[kpoints]
ksteps = [4, 4]      #Increments of wavenumber.
                    #If 0, half of lattice size instead of 0 is set.
```

K = 0: (0,0)
K = 1: (π ,0)
K = 2: (0, π)
K = 3: (π , π)

xmzsk

The longitudinal susceptibility

$$\chi^{zz}(\vec{k}, \omega = 0) = \beta N_s [\langle (\tilde{m}_K^z)^2 \rangle - \langle \tilde{m}_K^z \rangle^2]$$

where
$$m_K^z = \frac{1}{N_s} \sum_i^{N_s} M_i^z \cos(\mathbf{k} \cdot \mathbf{r}_i)$$

Tutorial: Number density of the hardcore Bosons on a square lattice (5)

- **Modify `exec.py`**

```
import subprocess

from dsqss.dla_pre import dla_pre
from dsqss.result import Results

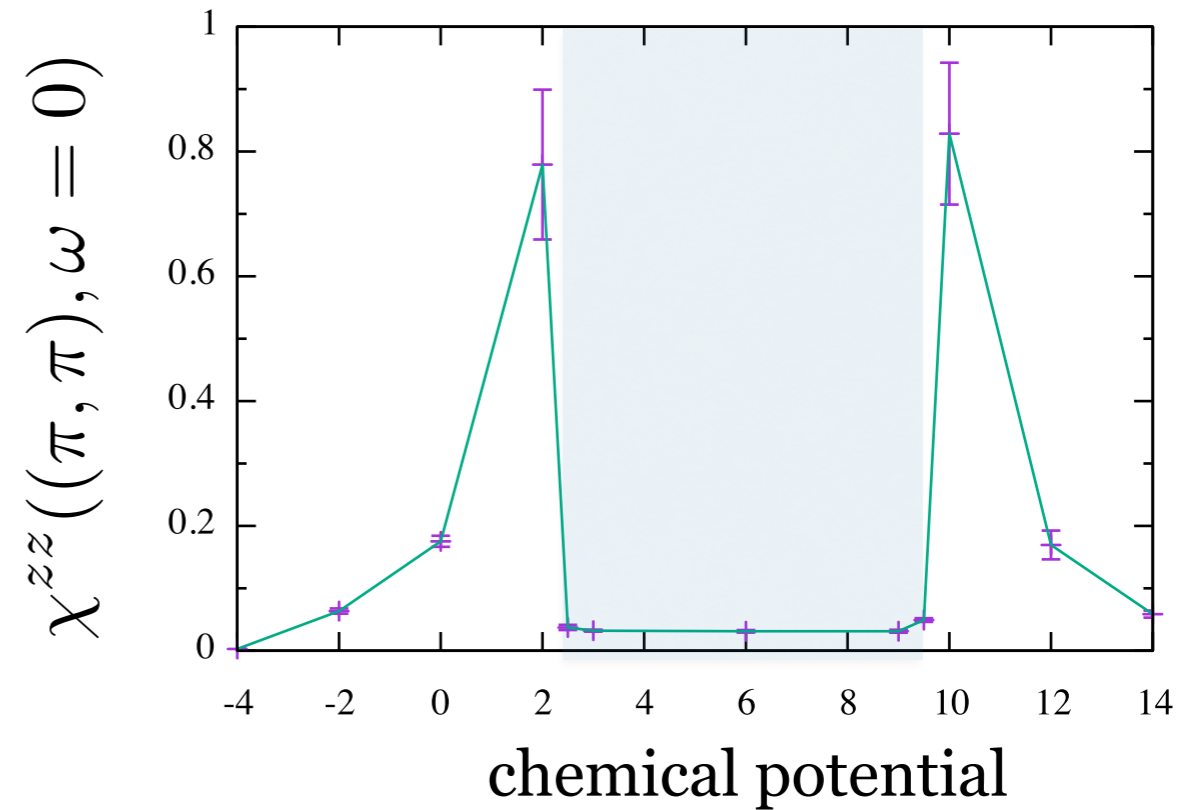
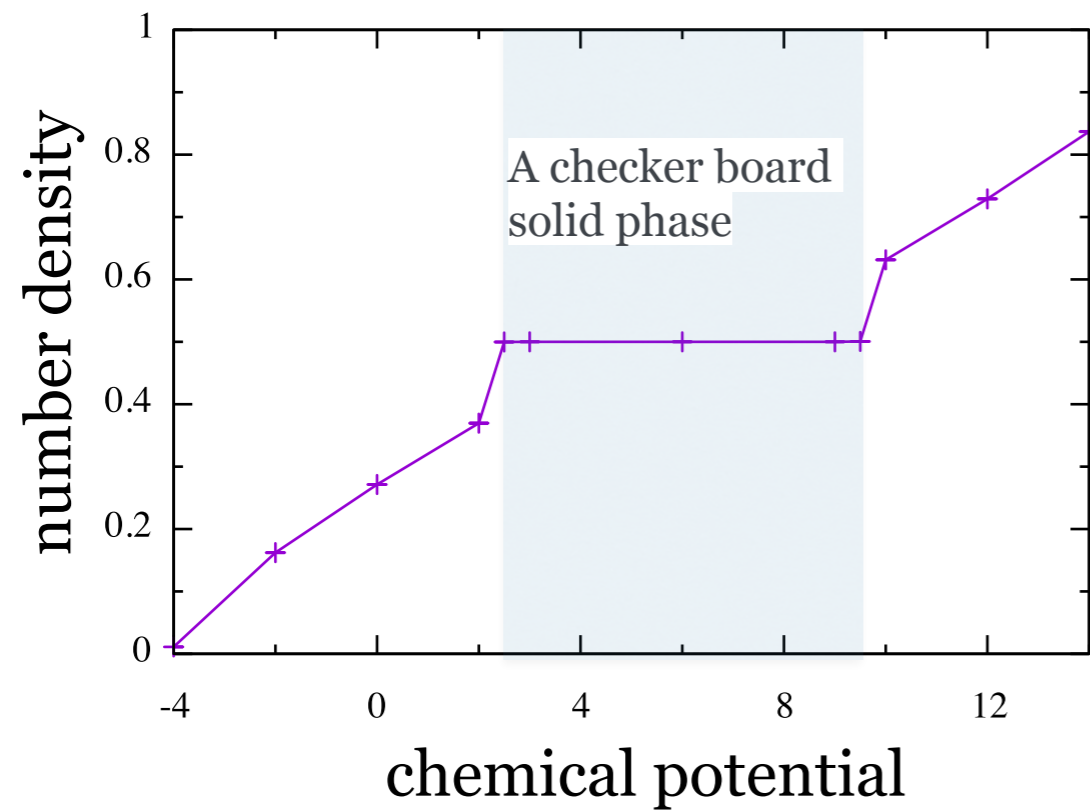
V = 3
L = [8, 8]
beta = 10.0

lattice = {"lattice": "hypercubic", "dim": 2, "L": L}
hamiltonian = {"model": "boson", "t": 1, "V": V, "M": 1}
parameter = {"beta": beta, "nset": 4, "ntherm": 100, "ndecor": 100, "nmcs": 100}

name = "smzs3"
mus = [-4.0, -2.0, 0.0, 2.0, 2.5, 3.0, 6.0, 9.0, 9.5, 10.0, 12.0, 14.0]

output = open("{} .dat".format(name), "w")
for i, mu in enumerate(mus):
    ofile = "res_{}.dat".format(i)
    pfile = "param_{}.in".format(i)
    wfile = "wave_{}.out".format(i)
    hamiltonian["mu"] = mu
    parameter["outfile"] = ofile
    parameter["wvfile"] = wfile
    dla_pre(
        {"parameter": parameter, "hamiltonian": hamiltonian, "lattice": lattice}, pfile
    )
    cmd = ["dla", pfile]
    # subprocess.call(cmd)
    res = Results(ofile)
    output.write("{} {} \n".format(mu, res.to_str(name)))
output.close()
```

Tutorial: Number density of the hardcore Bosons on a square lattice (6)



Other output files

1. Structure factor (Keyword: sfoutfile in std.tom)

$$S^{zz}(\vec{k}, \tau) \equiv \langle M^z(\vec{k}, \tau) M^z(-\vec{k}, 0) \rangle - \langle M^z(\vec{k}, \tau) \rangle \langle M^z(-\vec{k}, 0) \rangle$$

Wave vector k and imaginary time t are specified by the name `C<k>t<t>` as the following:

```
R C0t0 = 1.325000000e-03 1.40929454e-04  
R C0t1 = 1.325000000e-03 1.40929454e-04  
R C1t0 = 7.35281032e-02 3.18028565e-04
```

2. Real temperature Green's function (Keyword: cfoutfile in std.toml)

$$G(\vec{r}_{ij}, \tau) \equiv \langle M_i^+(\tau) M_j^- \rangle$$

3. Momentum space temperature Green's function (Keyword: ckoutfile in std.toml)

$$G(\vec{k}, \tau) \equiv \langle M^+(\vec{k}, \tau) M^-(-\vec{k}, 0) \rangle$$

Detail of input file

1. [parameter] section

A table specifying simulation parameters such as the inverse temperature.

2. [hamiltonian] section

A table specifying information of Hamiltonian.

3. [lattice] section

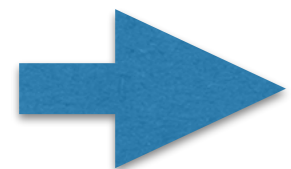
A table specifying information of lattice.

4. [kpoints] section

A table specifying information of wavevectors.

5. [algorithm] section

A table specifying algorithm for calculating scattering probability of wormheads



See details in Sec. 4.1 「Simple mode of DSQSS/DLA」 .

Detail of input file

1. [parameter] section

A table specifying simulation parameters such as the inverse temperature.

2. [hamiltonian] section

A table specifying information of Hamiltonian.

3. [lattice] section

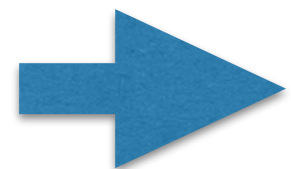
A table specifying information of lattice.

4. [kpoints] section

A table specifying information of wave vectors.

5. [algorithm] section

A table specifying algorithm for calculating scattering probability of wormheads



See details in Sec. 4.1 「Simple mode of DSQSS/DLA」 .

Detail of input file (1) Hamiltonian

XXZ model for arbitrary S

$$\mathcal{H} = - \sum_{\langle i,j \rangle} \left[J_z S_i^z S_j^z + \frac{J_{xy}}{2} (S_i^+ S_j^- + S_i^- S_j^+) \right] + D \sum_i (S_i^z)^2 - h \sum_i S_i^z$$

The value of S on a site can be specified by the keyword $M (=2S)$ in the input file.

Bose-Hubbard model

$$\mathcal{H} = - \sum_{\langle i,j \rangle} \left[t(b_i^\dagger b_j + \text{h.c.}) + V n_i n_j \right] + \sum_i \left[\frac{U}{2} n_i (n_i - 1) - \mu n_i \right]$$

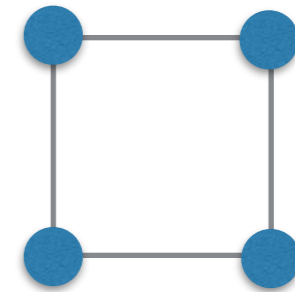
The cutoff of the number of particles on a site can be specified by the keyword M in the input file.

Detail of input file (2) Lattice

1. hypercubic

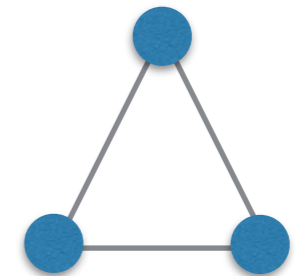
A hyper cubic lattice with arbitrary dimension.

By using `bc`, users can generate ladder or slab lattices.



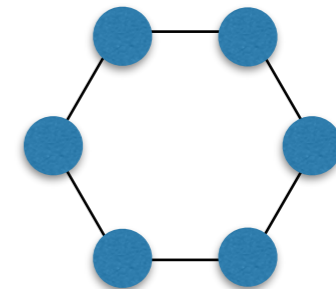
2. triangular

A two dimensional triangular lattice.



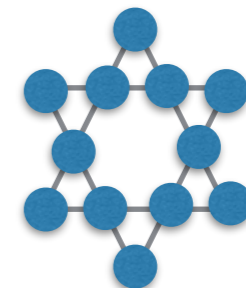
3. honeycomb

A two dimensional honeycomb lattice.



4. kagome

A two dimensional kagome lattice.



More complex lattice / Hamiltonian \rightarrow Standard mode